

Sum of Numbers Problems.

Sum of Numbers type problems take the form *'The sum of three consecutive odd integers is 99. What are the numbers?'* Depending on the problem the numbers may be odd, they may be even or they may be consecutive- both odd and even. To get some clue as to how we might solve such a problem let us try to determine the pattern and then we will apply this pattern to our actual problem. To start, let us suppose we are tasked with creating such a problem for our students. Let's select three consecutive odd integers and let's make the first integer 7. Then the following two integers will be 9 and 11 as shown below:

7

9

11

Now the sum of these three consecutive odd integers is 27 so our created problem will read:

"The sum of three consecutive odd integers is 27. What are the integers?"

Now, we know what the answer is because we just created the problem but what is the pattern? Well, there are a couple things we can see about this example:

- (1) First, the integers differ from each other by a count of two so, if the first integer is x then the second integer is $x+2$ and the third integer is $x+4$. So, algebraically, the problem becomes $(x) + (x+2) + (x+4) = 27$. Now the parenthesis are not necessary; they are just put there to emphasize the page numbers in an algebraic form. We can remove the parenthesis, gather like terms and solve this problem as follows: $3x + 6 = 27$. So $3x = 21$ and $x = 7$ which, of course, is the number we selected to start our exercise, and the other two numbers are 9 and 11. Now this is a pretty general structure and this pattern can now be applied to any of these types of consecutive integer problems. The difference term, in this case 2, will still be 2 if the consecutive integers are even but the difference will be 1 if the numbers are just consecutive, both odd and even.
- (2) The second thing we might notice about this problem is that 27 divided by the number of integers, 3, is 9 and 9 is the middle number of the set. The other two numbers are two greater than 9, which is 11, and two less than 9 which is 7. Now this second observation is a little tricky and you would have to be very careful how it was applied since it won't work very well if there are an even number of integers, but it does work well if there are an odd number of integers. So I don't recommend using techniques like this for general case Sum of Numbers type problems even though it is much quicker than the algebraic approach.

In summary, this technique of pretending you are creating an algebraic problem similar to the problem you are trying to solve, except using your own numbers, is a very good way to discover the patterns in the problem. Once you have discovered the patterns then writing down the general case algebraic expressions becomes much easier.

